GCE A LEVEL MARKING SCHEME

SUMMER 2025

A LEVEL
FURTHER MATHEMATICS
UNIT 6 FURTHER MECHANICS B
1305U60-1

About this marking scheme

The purpose of this marking scheme is to provide teachers, learners, and other interested parties, with an understanding of the assessment criteria used to assess this specific assessment.

This marking scheme reflects the criteria by which this assessment was marked in a live series and was finalised following detailed discussion at an examiners' conference. A team of qualified examiners were trained specifically in the application of this marking scheme. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners. It may not be possible, or appropriate, to capture every variation that a candidate may present in their responses within this marking scheme. However, during the training conference, examiners were guided in using their professional judgement to credit alternative valid responses as instructed by the document, and through reviewing exemplar responses.

Without the benefit of participation in the examiners' conference, teachers, learners and other users, may have different views on certain matters of detail or interpretation. Therefore, it is strongly recommended that this marking scheme is used alongside other guidance, such as published exemplar materials or Guidance for Teaching. This marking scheme is final and will not be changed, unless in the event that a clear error is identified, as it reflects the criteria used to assess candidate responses during the live series.

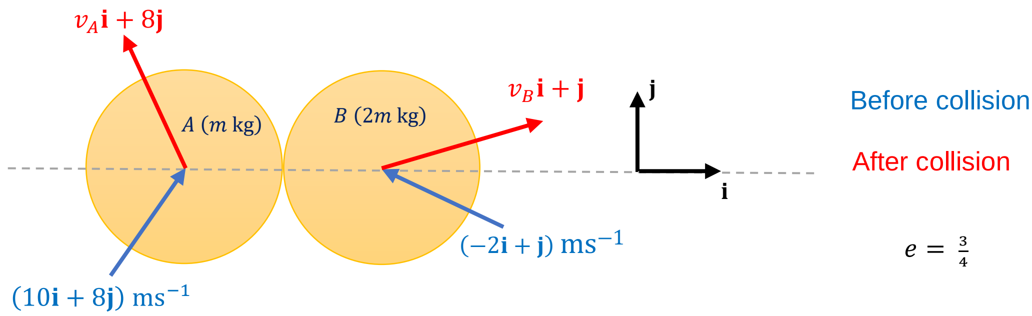
WJEC GCE A LEVEL FURTHER MATHEMATICS

UNIT 6 FURTHER MECHANICS B

SUMMER 2025 MARK SCHEME

Q1	Solution	Mark	Notes
(a)	(i) $F = \frac{96\times1000}{40}$ $(= 2400)$	B1	Sight of $F = \frac{96\,(000)}{v}$ in (i) or (ii), oe
	Using N2L at maximum speed with $a = 0$	M1	$F - kv^2 = 0$
	$\frac{96\times1000}{40} = k(40)^2$		
	$k = \frac{3}{2} = 1\cdot5$	A1	Convincing, intermediate line required
	(ii) Applying N2L (F, kv^2 opposing),	M1	Dimensionally correct equation $F - kv^2 = \pm ma$
	$\frac{96\,(000)}{v} - 1\cdot5v^2 = 2250a$	A1	Fully correct equation, oe
	Multiplying by v and using $a = v\frac{\mathrm{d}v}{\mathrm{d}x}$		
	$2250v\left(v\frac{\mathrm{d}v}{\mathrm{d}x}\right) = 96\,000 - 1\cdot5v^3$		
	$1500v^2\frac{\mathrm{d}v}{\mathrm{d}x} = 64000 - v^3$	A1	Convincing
		[6]	
(b)	$1500\int\frac{v^2}{64000-v^3}\,\mathrm{d}v = \int \mathrm{d}x$ $\quad \pm15(00)\int\frac{v^2}{64(000)-v^3}\,\mathrm{d}v = \pm\int \mathrm{d}x$	M1	Separating variables Integration notation not required
	$\frac{1500}{-3}\ln\lvert64\,000 - v^3\rvert = x \quad (+C)$	A1 A1	$\ln\lvert64\,000 - v^3\rvert$ Everything correct, oe $\lvert\quad\rvert$ not needed for both
	When $x = 0, v = 10$		
	$-\frac{1500}{3}\ln\lvert64000 - 10^3\rvert = C$	m1	Use of initial conditions (accept use of limits->examiner notes)
	$C = -500\ln\lvert63000\rvert$	A1	Finding C, FT $\pm C$ from sign error only $C \approx 5525\cdot445$ ...
	$x = 500\ln\lvert63000\rvert - 500\ln\lvert64000 - v^3\rvert$		
	$\frac{x}{500} = \ln\left\lvert\frac{63000}{64000 - v^3}\right\rvert$		
	$\frac{63000}{64000 - v^3} = \mathrm{e}^{\frac{x}{500}}$	m1	Inversion, FT $C \neq 0$
	$64000 - v^3 = 63000\mathrm{e}^{-\frac{x}{500}}$		
	$v^3 = 1000\left(64 - 63\mathrm{e}^{-\frac{x}{500}}\right)$	A1	oe, isw
		[7]	

Q1	Solution	Mark	Notes
(c)	When $x = 250$, $v^3 = 1000\left(64 - 63e^{-\frac{250}{500}}\right)$ $\quad(= 25\,788 \cdot 568\,\ldots)$ $v = 29 \cdot 5(444\,\ldots)$	M1 A1 **[2]**	Used in their expression for v^3 or x cao
(d)	Appropriate explanation, e.g. • When $v = 40$ (or $v^3 = 64\,000$) we get **division by zero** (in expression for x) OR we get an **undefined expression** (for x) • $v = 40$ is a limiting value	E1 **[1]**	
	Total for Question 1	16	

Q2	Solution	Mark	Notes
			
(a)	Con. of momentum (along line of centres)	M1	Attempted, allow 1 sign error.
	$m(10) + 2m(-2) = mv_A + 2m(v_B)$	A1	Consistency of v_A, v_B across M's All correct, oe
	$v_A + 2v_B = 6$		
	Restitution (along line of centres)	M1	Attempted, allow 1 sign error.
	$v_B - v_A = -\frac{3}{4}(-2 - 10)$	A1	All correct, oe
	$\frac{3}{4} = \frac{v_B - v_A}{10 - -2} = \frac{v_A - v_B}{-2 - 10}$ OR $-\frac{3}{4} = \frac{v_B - v_A}{-2 - 10} = \frac{v_A - v_B}{10 - -2}$		
	$v_B - v_A = 9$		
	Solving equations $v_A = -4$ $v_B = 5$	m1	One variable eliminated.
	Velocities after collision		
	Sphere $A = -4\mathbf{i} + 8\mathbf{j}$ (ms^{-1})	A1	cao, isw (i.e. magnitude)
	Sphere $B = 5\mathbf{i} + \mathbf{j}$ (ms^{-1})	A1	cao, isw
		[7]	
(b)	Impulse on B		
	$\mathbf{I} = m_B\mathbf{v}_B - m_B\mathbf{u}_B$	M1	Used,
	$= (2m)[(5\mathbf{i} + \mathbf{j}) - (-2\mathbf{i} + \mathbf{j})]$		FT $\mathbf{v}_B$
	$= 14m\mathbf{i}$ Ns	A1	**Units must be included,** cao Allow kg ms^{-1}
		[2]	
	Total for Question 2	**9**	

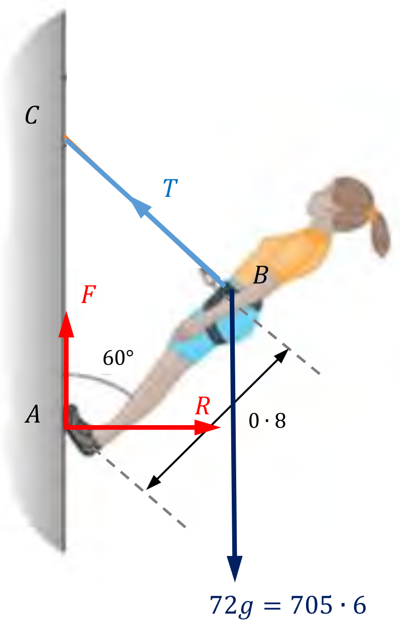
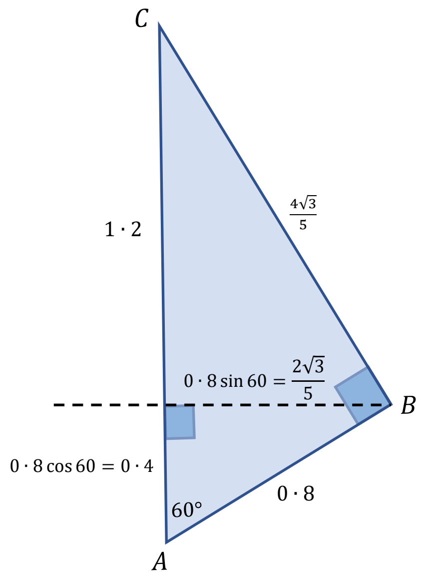
Q2	Solution	Mark	Notes
(a)	<u>Alternative Solution for M1A1 in (a)</u> Conservation of momentum $m\mathbf{u}_A + 2m\mathbf{u}_B = m\mathbf{v}_A + 2m\mathbf{v}_B$ $m(10\mathbf{i} + 8\mathbf{j}) + 2m(-2\mathbf{i} + \mathbf{j})$ $= m(v_A\mathbf{i} + 8\mathbf{j}) + 2m(v_B\mathbf{i} + \mathbf{j})$ $v_A + 2v_B = 6$	M1 A1	Attempted. All correct, oe
(b)	<u>Alternative Solution for (b)</u> Impulse on A $\mathbf{I} = m_A\mathbf{v}_A - m_A\mathbf{u}_A$ $= m[(-4\mathbf{i} + 8\mathbf{j}) - (10\mathbf{i} + 8\mathbf{j})]$ $= -14m\mathbf{i}$ $\|\mathbf{I}\| = 14m\mathbf{i}$ Ns	M1 A1 **[2]**	Used FT $\mathbf{v}_A$ **Units must be included,** cao Allow kg ms^{-1}
(b)	<u>Alternative Solution for (b): Non-vector versions</u> Impulse on B (Impulse on A) $I = m_B v_B - m_B u_B$ $\quad = m_A v_A - m_A u_A$ $= (2m)\big(5 - (-2)\big)$ $\quad = m(-4 - 10)$ $= 14m$ Ns $\quad = -14m$ Ns $\mathbf{I} = 14m\mathbf{i}$ Ns	M1 A1 **[2]**	Used FT ANY v_A, v_B **Units must be included,** cao The $\mathbf{i}$ MUST (re)appear OR direction must be given Allow kg ms^{-1}

Shape	Area	Mass	Distance from AG	Distance from AD
ADFG	60×120 $(= 7200)$	7200ρ	60	30
Quad AHB	$\dfrac{\pi(30)^2}{4}$ $(= 225\pi)$	$\pm 225\pi\rho$	$\dfrac{40}{\pi}$	$\dfrac{40}{\pi}$
Quad DCE	$\dfrac{\pi(30)^2}{4}$ $(= 225\pi)$	$\pm 225\pi\rho$	$120 - \dfrac{40}{\pi}$	$\dfrac{40}{\pi}$
L	225π	$2 \times 225\pi\rho$	80	a
Sign		$7\,200\rho$	$\bar{x}$	$\bar{y}$

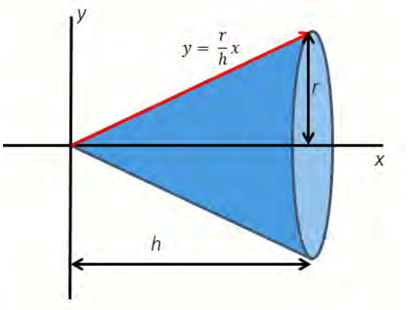
Q3	Solution	Mark	Notes
(a)	(Table above)	B3	Condone • omission of ρ (mass per unit area) • inclusion of g All 12 values correct B2 any 2 rows or columns correct B1any row or column correct $\frac{4(30)}{3\pi} = \frac{40}{\pi} = 12 \cdot 732\ldots$ $120 - \frac{40}{\pi} = 107 \cdot 2676\ldots$
	Sign row	B1	Correct combination of masses (ADFG - QUADx2 + L) FT their values.
	$\bar{y} = 35$ (i) Moments about AG	M1	Masses and moments consistent All terms, allow sign errors. FT table.
	$7200\bar{x} = (7200)(60) - (225\pi)\left(\frac{40}{\pi}\right) - (225\pi)\left(120 - \frac{40}{\pi}\right) + (450\pi)(80)$ $7200\bar{x} = 432000 - 9000 - 27000\pi + 9000 + 36000\pi$ $7200\bar{x} = 432000 + 9000\pi \quad = 460274 \cdot 33..$	A1	FT equations leading to $(k\bar{x}) = l + m\pi$ (ADFG - QUADx2 + L)
	$\bar{x} = 60 + \frac{5}{4}\pi = 63 \cdot 926(9908\ldots) \quad (\text{cm})$	A1	cao, any form
	(ii) Moments about AD (use of $\bar{y}$ for A1 only)	M1	Masses and moments consistent All terms, allow sign errors. FT table.
	$(7200)(35) = (7200)(30) - (225\pi)\left(\frac{40}{\pi}\right) - (225\pi)\left(\frac{40}{\pi}\right) + (450\pi)(a)$ $252000 = 216000 - 9000 - 9000 + 450\pi a$	A1	FT equations leading to $(k) = l + \pi a$ (ADFG - QUADx2 + L)
	$a = \frac{120}{\pi} = 38 \cdot 19(718634\ldots) \quad (\text{cm})$	A1	cao, any form
		[10]	
(b)	Length $BP = \bar{x} - 30$ $= 30 + \frac{5}{4}\pi = 33 \cdot 92\ldots \quad (\text{cm})$	B1	FT $\bar{x}$ from (a)(i)
		[1]	
	Total for Question 3	**11**	

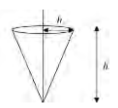
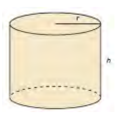
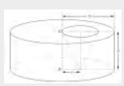
Q4	Solution	Mark	Notes
(a)	(i) $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\omega^2 x \quad \Leftrightarrow \quad v\frac{\mathrm{d}v}{\mathrm{d}x} = -\omega^2 x$		
	$\int v\,\mathrm{d}v = -\omega^2 \int x\,\mathrm{d}x$	M1	$a = v\frac{\mathrm{d}v}{\mathrm{d}x}$ used and separation of variables, ALLOW $\pm$
	$\frac{1}{2}v^2 = -\omega^2 \times \frac{1}{2}x^2 \ (+C)$	A1	
	When $x = a, v = 0$ $C = \frac{1}{2}\omega^2 a^2$	m1	Use of initial conditions
	$v^2 = \omega^2(a^2 - x^2)$	A1	Convincing
	(ii) $v = \frac{\mathrm{d}x}{\mathrm{d}t} = \omega\sqrt{(a^2 - x^2)}$	M1	Allow $\pm\sqrt{\omega^2(a^2 - x^2)}$ $v =$ or $\frac{\mathrm{d}x}{\mathrm{d}t} =$
	$\int \frac{1}{\sqrt{(a^2 - x^2)}}\mathrm{d}x = \omega \int \mathrm{d}t$	M1	$v = \frac{\mathrm{d}x}{\mathrm{d}t}$ used and separation of variables, $\pm$
	$\sin^{-1}\left(\frac{x}{a}\right) = \omega t \ (+C)$	A1	
	When $t = 0, x = 0$ $C = 0$	m1	
	$x = a\sin(\omega t)$	A1	Convincing
		[9]	
(b)	(i) $v^2 = \omega^2(a^2 - x^2)$	M1	Used
	$8^2 = \omega^2(a^2 - 3^2)$ $6^2 = \omega^2(a^2 - 4^2)$	A1	Either correct
	Eliminating one variable $\omega = 2$ $a = 5$	m1 A1 A1	 cao Convincing
	Period, $T = \frac{2\pi}{\omega} = \pi \quad (\mathrm{s})$	A1	cao
	(ii) Using $x = \pm 5\sin(2t)$ with $t = 1\cdot 8$	M1	FT ω
	At $t = 1\cdot 8$, $x = 5\sin(2 \times 1\cdot 8)$ $= -2\cdot 2126 \ldots$		
	Distance $= 2\cdot 21(26\ldots) \quad (\mathrm{m})$	A1	cao
		[8]	
	Total for Question 4	**17**	

Q4	Solution	Mark	Notes
	<u>Alternative Solution to (a) (ii)</u>		
	Auxiliary equation: $r^2 + \omega^2 = 0$ Roots: $r = \pm\omega\mathrm{i}$	M1 A1	
	General solution: $x = \mathrm{e}^{-ot}(A\cos\omega t + B\sin\omega t)$	B1	oe
	Initial conditions: $t = 0,\ x = 0$ $(\Rightarrow A = 0)$		
	$x = B\sin\omega t$		
	$v = B\omega\cos\omega t$	m1	$t = 0\ \ x = 0$ used in expression for x and x' attempted
	Conditions: $v = 0$ when $x = a$ $0 = B\omega\cos\omega t \quad \Rightarrow \quad \omega t = \frac{\pi}{2}, \ldots$ $\therefore \quad a = B\sin\frac{\pi}{2} \quad \Rightarrow \quad a = B$		**Not using** $v = 0$ when $x = a$ $\omega\sqrt{(a^2 - x^2)} = B\omega\cos\omega t$ Using $t = 0,\ x = 0$ $\sqrt{(a^2 - 0^2)} = B\cos 0 \quad \Rightarrow \quad a = B$
	$\therefore \quad x = a\sin\omega t$	A1	Convincing
	Total for Question 4 (a) (ii)	5	

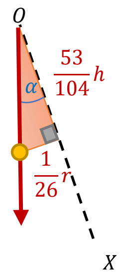
Q5	Solution	Mark	Notes
(a)	 		
	Moments about A	M1	Dim. correct equation, no extra terms Allow $\cos 60°$
	$T \times 0\cdot8 = 72g \times 0\cdot8\sin 60°$	A1	oe
	$T = 36\sqrt{3}g = \frac{1764\sqrt{3}}{5} = 611\cdot0(675249\,...)$ (N)	A1 **[3]**	cao, any form
(b)	Finding the values of F and R		M's: Allow trig errors
	Resolve vertically $F + T\cos 30° = 72g$ F pointing downwards ($T\cos 30° = F + 72g$)	M1 A1	Dim. correct equation, no extra/missing forces.
	Resolve horizontally or Moments about C $T\sin 30° = R$ or $R \times 1\cdot6 = 72g \times 0\cdot8\sin 60°$	M1 A1	Dim. correct equation, no extra forces
	Substitution of T or attempting to evaluate R	m1	Sub. of T into at least one equation
	$F = 18g = 176\cdot4 = \frac{882}{5}$ (N) $R = 18\sqrt{3}g = \frac{882\sqrt{3}}{5} = 305\cdot5(337625)$ (N)	A1	cao, at least one correct
	Limiting Friction; use of $F = \mu R$	M1	$F \le \mu R$ si, sub in their F OR R
	$\mu = \frac{F}{R} = \frac{18g}{18\sqrt{3}g} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} = 0\cdot577(3502692)$	A1 **[8]**	cao
(c)	In reality, any of the given modelling assumptions may not be true.	B1 **[1]**	
	Total for Question 5	12	

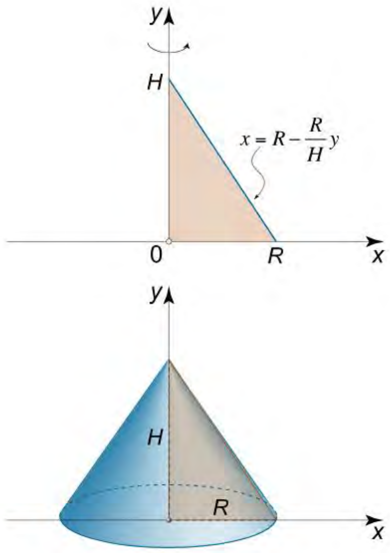
Q5	Solution	Mark	Notes
(b)	Alternative Solution to (b) Elimination of F using $F = \mu R$ Resolve vertically — F pointing downwards $F + T\cos 30° = 72g$ — ($T\cos 30° = F + 72g$) Resolve horizontally or Moments about C $T\sin 30° = R$ or $R \times 1\cdot 6 = 72g \times 0\cdot 8\sin 60°$ $R = 18\sqrt{3}g = 305\cdot 5(337625)$ (N) Limiting Friction; use of $F = \mu R$ $\mu R + T\cos 30° = 72g$ $\mu = \frac{72g - T\cos 30°}{T\sin 30°}$ Substitution of T to get $\mu = \frac{F}{R} = \frac{18g}{18\sqrt{3}g} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} = 0\cdot 577(3502692)$	 M1 A1 M1 A1 A1 M1 m1 A1 **[8]**	M's: Allow trig errors Dim. correct equation, no extra/missing forces, Dim. correct equation, no extra forces cao FT R Sub. of T into at least one equation overall cao, any form
(b)	Alternative Solution to (b) Moments about B Moments about B $F \times 0\cdot 8\sin 60° = R \times 0\cdot 8\cos 60°$ $\mu = \frac{F}{R} = \frac{1}{\tan 60°} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} = 0\cdot 577(3502692)$	 M1 A1 A1 M1 A4 **[8]**	 Dim. correct equation, no extra terms Allow trig errors LHS RHS $F = \mu R$ used cao, any form

Q6	Solution	Mark	Notes
(a)			$y = \begin{cases} \frac{r}{h}x \\ r - \frac{r}{h}x = r\left(1 - \frac{1}{h}x\right) \end{cases}$
	Rotation about the x-axis		
	$(V\bar{x} =)\ \pi \int_0^h xy^2\,dx$	M1	Used with $y = f(x) = \pm ax \pm b, a \neq 0$ π si, y^2 could be incorrect
	$(V\bar{x}) = \pi \int_0^h x\left(\frac{r}{h}x\right)^2 dx \qquad = \frac{r^2}{h^2}\pi \int_0^h x^3\,dx$	A1	All correct, oe, limits not needed $(V\bar{x}) = \pi \int_0^h x\left(r - \frac{r}{h}x\right)^2 dx$
	$(V\bar{x}) = \frac{r^2}{4h^2}\pi[x^4]_0^h$		
	$(V\bar{x}) = \frac{1}{4}\pi r^2 h^2$	A1	$(V\bar{x}) = \frac{1}{12}\pi r^2 h^2$
	Using $V = \frac{1}{3}\pi r^2 h$ and dividing to get $\bar{x} = \frac{3}{4}h$		$\bar{x} = \frac{1}{4}h$
	Distance from base $= h - \frac{3}{4}h = \frac{1}{4}h$	A1 **[4]**	Distance from base $= \frac{1}{4}h$ Convincing
(b)	(table below)	B2	Condone • omission of ρ (mass per unit area) • inclusion of g All 6 entries correct B1 One correct row or column

Shape	Volume/Mass	Distance from OX	Distance from top plane face
	$\pm\frac{1}{3}\pi r^2 h \quad (\times \rho)$ $(= \frac{1}{3}\pi r^2 h\rho)$	r	$\frac{1}{4}h$
	$\pi(3r)^2 \times h \quad (\times \rho)$ $(= 9\pi r^2 h\rho)$	0	$\frac{1}{2}h$
	$9\pi r^2 h - \frac{1}{3}\pi r^2 h\ (\times \rho)$ $\left(= \frac{26}{3}\pi r^2 h\rho\right)$	$\bar{x}$	$\bar{y}$

		B1	Difference of volume/mass FT above B1's $\left(= \frac{k}{l}\pi r^2 h\rho\right)$
	(i) Moments about top plane face	M1	No extra/missing terms. Allow sign errors
	$\frac{26}{3}\pi r^2 h\rho \times \bar{y} = 9\pi r^2 h\rho \times \frac{1}{2}h - \frac{1}{3}\pi r^2 h\rho \times \frac{1}{4}h$	A1	FT table, each term of the form $\frac{k}{l}\pi r^2 h\rho$
	$\frac{26}{3}\pi r^2 h \times \bar{y} = \frac{53}{12}\pi r^2 h^2$		Note that $\frac{1}{4}h \to \frac{3}{4}h$ when working relative to bottom plane face, $\bar{y} = h - \frac{51}{104}h$
	$\bar{y} = \frac{53}{104}h$	A1 **[6]**	Convincing

Q6	Solution	Mark	Notes
	(ii) Moments about OX $\frac{26}{3}\pi r^2 h\rho \times \bar{x} = 9\pi r^2 h\rho \times 0 - \frac{1}{3}\pi r^2 h\rho \times r$ $\bar{x} = -\frac{1}{26}r$ Distance = $\frac{1}{26}r$	M1 A1 A1 **[3]**	No extra/missing terms. Allow sign errors FT table, each term of the form $\frac{k}{l}\pi r^2 h\rho$ cao, accept $\bar{x} = \pm\frac{1}{26}r$
	(iii) If hanging in equilibrium, vertical passes through centre of mass.  $\tan\alpha = \left(\frac{\frac{1}{26}r}{\frac{53}{104}h} = \right)\frac{104r}{1378h} = \frac{4r}{53h}$	M1 A1 **[2]**	Correct triangle identified $\sin/\cos/\tan\alpha = \frac{\bar{x}}{\frac{53}{104}h} = \frac{\frac{53}{104}h}{\bar{x}}$ or sight of a supporting diagram ($\perp$ triangle) with the sides $\frac{53}{104}h,\ \bar{x}$ cao, accept $\pm$
	Total for Question 6	15	

Q6	Solution	Mark	Notes
(a)	Alternative Solution to (a)		
			$x = \begin{cases} \frac{r}{h}y \\ r - \frac{r}{h}y = r\left(1 - \frac{1}{h}y\right) \end{cases}$
	Rotation about the y-axis		
	$(V\bar{y} =)\ \pi \int_0^h x^2 y \, \mathrm{d}y$	M1	Used with $x = f(y)$
	$(V\bar{y}) = \pi \int_0^h \left(\frac{r}{h}y\right)^2 y \, \mathrm{d}y \qquad = \frac{r^2}{h^2}\pi \int_0^h y^3 \, \mathrm{d}y$	A1	All correct, oe, limits not needed $(V\bar{y}) = \pi \int_0^h y\left(r - \frac{r}{h}y\right)^2 \mathrm{d}y$
	$(V\bar{x}) = \frac{r^2}{4h^2}\pi[y^4]_0^h$		
	$(V\bar{y}) = \frac{1}{4}\pi r^2 h^2$	A1	$(V\bar{y}) = \frac{1}{12}\pi r^2 h^2$
	Using $V = \frac{1}{3}\pi r^2 h$ and dividing to get		
	$\bar{y} = \frac{3}{4}h$		$\bar{y} = \frac{1}{4}h$
	Distance from base $= h - \frac{3}{4}h = \frac{1}{4}h$	A1 **[4]**	Distance from base $= \frac{1}{4}h$ Convincing